The Dominant Mode Properties of Open Groove Guide: An Improved Solution

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Abstract —**Groove guide, one of several low-loss waveguides proposed some years ago for use at millimeter wavelengths, is again receiving attention in the literature. A new transverse equivalent network and dispersion relation for the properties of the dominant mode are presented here which are extremely simple in form and yet very accurate. Comparisons with accurate published measurements indicate better agreement with this new theory than with any previous theory.**

I. Introduction

A. Background

GROOVE GUIDE is one of a group of waveguiding structures proposed some 20 or more years ago for use at millimeter wavelengths. Those waveguides were not pursued beyond some initial basic studies because they were not yet needed, and because adequate sources for millimeter waves were not yet available. Today, such sources are readily available, and the many advantages of millimeter waves are becoming increasingly appreciated.

It was also recognized some years ago that the shorter wavelengths associated with millimeter waves produce problems relating to the small size of components and the high attenuation of waveguides. New types of waveguide were therefore proposed for which the attenuation per unit length would be substantially lower than that for customary waveguides, and for which, in some cases, the cross-section dimensions were greater. For components which are only a wavelength or so long, somewhat lossier waveguides are acceptable, and indeed waveguides such as microstrip and finline are being employed successfully in this connection, particularly at the longer wavelength end of the millimeter-wave range. For long runs of waveguide, however, and for certain components such as leaky-wave antennas, for example, which may typically be 20 λ_0 to 100 λ_0 in length, where λ_0 is the free-space wavelength, the intrinsic attenuation of a lossy waveguide would compete with the leakage loss corresponding to the radiation, and it could seriously interfere with the antenna performance. Therefore, attention is again being paid to new types of low-loss waveguide, one of which is groove guide.

Results for the propagation characteristics of the dominant mode in groove guide have been published previously. There exist theoretical expressions which are simple but approximate, more accurate expressions which involve infinite sums and are messy to compute from, and careful measured results. We present in this paper a *new expression* for the propagation constant of groove guide, which is *very accurate*, *yet in closed form and simple*. A derivation is presented of the new expression, and then comparisons are made with previously published theoretical and experimental results. It will be seen that the new expression provides *excellent agreement with measurement*, and in fact better agreement than with any previous theoretical data.

The motivation for obtaining an improved expression for the propagation constant of groove guide, and in the process a transverse equivalent network which is simple and whose constituents are all in closed form, is that groove guide appears to be an excellent low-loss waveguide upon which can be based a number of novel leaky-wave antennas for the millimeter wavelength range. The results of this paper then form an important step in the analysis of such antennas. One antenna in this class has been described recently [1], [2]. It should be added that, in view of the small size of waveguiding structures at millimeter wavelengths, leaky-wave antennas form a *natural* class of antennas for these wavelengths.

B. The Properties of Groove Guide

The cross section of groove guide is shown in Fig. 1, and an indication of the dominant-mode electric field lines present in its cross section is given in Fig. 2(a). One should first note that the structure resembles that of rectangular waveguide with most of its top and bottom walls removed. The groove guide can therefore be excited by providing a smooth tapered transition between it and a feed rectangular waveguide. Furthermore, if symmetry is maintained, many components can be designed for groove guide which are analogs of those in rectangular guide.

With respect to the low-loss nature of groove guide, we should recall that when the electric field is parallel to the metal walls the attenuation associated with those walls decreases as the frequency is increased; conversely, the

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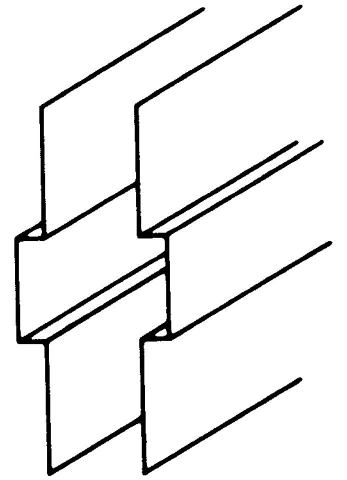


Fig. 1. The open groove guide, comprised of two parallel metal plates whose central regions are grooved outwards.

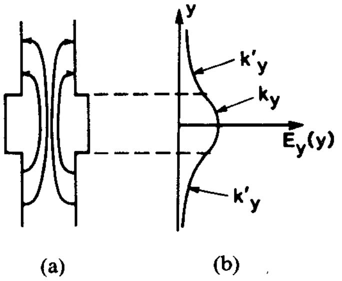


Fig. 2. The electric field of the dominant mode in open groove guide. (a) A sketch of the electric field lines in the cross section. (b) An approximate plot of the vertical component E_y as a function of vertical position y, showing that the guided mode is bound transversely to the central grooved region.

attenuation increases with increasing frequency when the electric field is perpendicular to the walls. Since, in groove guide, the electric field is seen to be mostly parallel to the walls, its overall attenuation at higher frequencies is much lower than that of rectangular waveguide, where most of the field is perpendicular at the top and bottom walls.

The greater width in the middle, or central, region was shown by T. Nakahara [3]–[5] to serve as the mechanism that confines the field in the vertical direction, much as the dielectric central region does in H guide. The field thus decays exponentially away from the central region in the narrower regions above and below, as shown in Fig. 2(b). If the narrower regions are sufficiently long, it does not matter if they remain open or are closed off at the ends. One may therefore regard the change from rectangular waveguide to groove guide as involving the replacement of most of the top and bottom walls in rectangular waveguide by reactive walls.

Work on the groove guide progressed in Japan [6], [7] and in the United States [8]–[10] until the middle 1960's, but then stopped and was later revived and developed further by D. J. Harris and his colleagues [11], [12] in Wales. The recent work is mainly experimental, being associated with components for groove guide.

The theoretical approach to the propagation constant of the dominant mode taken by most of the previous investigators [4]–[7], [9] has been to produce a first-order result by taking only the dominant transverse mode in each region of the cross section, and then obtaining the dispersion relation on use of the transverse resonance condition. That procedure, which neglects the presence of all higher transverse modes, is equivalent to accounting for the step junction between the central and outer regions by employing a transformer only, and by ignoring the junction susceptance entirely. With that approximation, a simple dispersion relation is obtained, which produces reasonably good agreement with measured data when the step discontinuity is small. More accurate theoretical phrasings were presented in [4]–[7] by accounting for the susceptance by taking an infinite number of higher modes on each side of the step junction and then mode matching at the junction. The resulting expressions involve matrices which, even after the necessary truncation, are messy to compute from. When only one or two higher modes are included, as in [6] and [7], the improvement in accuracy is quite small, and the added complexity in calculation is substantial.

The approach in this paper is to establish a proper transverse equivalent network, identify the appropriate transverse mode (which is hybrid), obtain an accurate expression in closed form for the step-junction susceptance, and then apply the transverse resonance condition to the now-complete transverse equivalent network, which yields the relevant dispersion relation for the propagation constant. This dispersion relation is simple, in closed form, and very accurate, as demonstrated in Section III, by comparison with measured data from [4] and [5].

II. The Transverse Equivalent Network

The complete transverse equivalent network for the groove guide is derived in this section by starting with a proper phrasing of the problem and then by putting together all the constituent elements. From this network, which characterizes the cross section of the guide, we finally obtain, via the transverse resonance condition, a dispersion relation for the propagation constant which is in simple closed form and yet accurate. The essential new constituent in the transverse equivalent network is a simple closed-form expression for the step-junction susceptance.

A. *Transverse Resonance Approach*

The general transverse resonance approach to deriving the propagation characteristics of a waveguiding structure is to obtain first a *transverse equivalent network* descriptive of the guide's cross section. That network is based on a *building-block* approach, in which uniform waveguide regions in the cross section are represented by transmission lines, and junctions or other discontinuities are represented by lumped elements. By inspection of the field configurations in the respective regions of the cross section, one then identifies the correct modes which the transmission lines represent, and then obtains the appropriate mode functions for those modes and the proper characteristic impedances for the transmission lines. The lumped elements corresponding to the discontinuities can be recognized in some

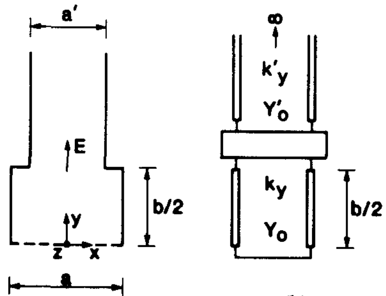


Fig. 3. Simplifications produced by short-circuit bisection. (a) Cross section of groove guide. (b) Form of transverse equivalent network.

instances, but need to be derived in other cases. When all the constituents of the network are known, a dispersion relation for the propagation characteristics of the waveguide is obtained by application of the transverse resonance condition.

A simplification in the transverse equivalent network is immediately available here by taking symmetry into account. By inspection of the structure in Fig. 1 and the field lines in Fig. 2(a), it is evident that the cross section can be bisected in short-circuit fashion, and that the resulting cross section and the corresponding form of the transverse equivalent network become the simplified ones shown in Fig. 3(a) and (b), respectively.

We must first identify the transverse mode which each transmission line represents, and correspondingly determine how the characteristic admittances Y_0 and Y_0' relate to the transverse wavenumbers k_y and k_y'. These relations are discussed in the next section, together with the transverse mode functions. Since the longitudinal mode under investigation is the dominant mode of groove guide, we recognize that the transverse wavenumbers in the x-direction are

$$k_x = \frac{\pi}{a} \quad \text{and} \quad k_x' = \frac{\pi}{a'} \tag{1}$$

in the central region of height b and in the outer regions, respectively, so that the various transverse wavenumbers are related to the longitudinal (propagation) wavenumber $k_z = \beta$ by the sum of the squares relations

$$\beta^2 = k_0^2 - k_y^2 - (\pi/a)^2 \tag{2}$$

$$\beta^2 = k_0^2 - k_y'^2 - (\pi/a')^2. \tag{3}$$

The free-space wavelength λ_0, the guide wavelength λ_g of the longitudinal mode, and the cutoff wavelength λ_c of the longitudinal mode are given by

$$\lambda_0 = 2\pi/k_0, \quad \lambda_g = 2\pi/\beta, \quad \lambda_c = 2\pi/k_t \tag{4}$$

where the total transverse wavenumber k_t is

$$k_t^2 = k_y^2 + (\pi/a)^2 = k_y'^2 + (\pi/a')^2. \tag{5}$$

Since the cross section contains only a single dielectric medium, the total transverse wavenumber k_t is a constant *independent of frequency*.

Since the guided dominant longitudinal mode is nonradiating, k_y is real and k_y' must be imaginary; that is, the unprimed (central) transmission line is above cutoff transversely and the primed (outer) transmission line is below cutoff transversely. (Those statements represent the facts that the fields in the central region of the groove guide are *transversely* propagating, and that those in the outer regions are transversely evanescent, as shown in Fig. 2(b). However, propagation occurs *longitudinally* in all parts of the cross section, of course.) If the below-cutoff primed line is sufficiently long, it does not matter how the line is terminated. On the other hand, if the narrower outer sections of the groove guide are closed by metal plates, and the outer sections (of length c, say) are not sufficiently long, the primed transmission line in the network of Fig. 3(b) must be terminated by a short circuit after a length c of transmission line. This "closed" groove guide case has been treated in detail (in a different way) in [6] and [7].

The box in Fig. 3(b) represents the step-junction discontinuity, and it is discussed in detail in Section II-C. The treatment of the step junction in this paper represents the basis for our new contribution.

After all the constituent portions of Fig. 3(b) have been properly characterized, the dispersion relation for the dominant mode is found from the lowest resonance of this network. That step is discussed in Section II-D.

B. *The Transverse Mode Functions*

To properly characterize the transmission lines in the transverse equivalent network of Fig. 3(b), we must identify the correct mode in the y-direction. We first note that with respect to the z- (longitudinal) direction, the overall guided mode is a TE (or H) mode; that is, there exists only a component of H in the z-direction. This result is to be expected since the groove guide consists of a perfectly conducting outer structure filled with only a single dielectric material (air). In the y-direction, however, there exist both E_y and H_y components, so that the mode is *hybrid* in that direction.

Since the groove guide is *uniform* in the z-direction, and its field has only an H_z component, the hybrid mode in the y-direction is seen to be what is called by some an H-type mode with respect to the z-direction, and by others an LSE mode with respect to the z-direction. We prefer the former notation, and we shall designate the mode in the y-direction as an $H^{(z)}$*-type mode*. Altschuler and Goldstone [13] discuss such modes in detail and present the field components for them and the characteristic admittances for transmission lines representative of them.

For the transmission lines in Fig. 3(b), we therefore require the mode functions and transmission-line properties of an $H^{(z)}$-type mode in parallel-plate guide, which propagates in the y-direction and is hybrid in that direction, but has only an H_z component in the z-direction. The coordinate system is that given in Fig. 3(a), but it differs by a rotation from the one employed in [13]. For our mode of interest, for which $k_x = \pi/a$, we find that the characteristic admittance is given by

$$Y_0 = \frac{k_0^2 - k_z^2}{\omega\mu k_y} \tag{6}$$

where k_y is the propagation constant of the transmission line. Consistent with this specification, the electric and magnetic field vector mode functions $\boldsymbol{e}$ and $\boldsymbol{h}$ satisfy the orthonormality condition

$$\int_s \boldsymbol{h} \times \boldsymbol{y}_0 \cdot \boldsymbol{e}^* \, dS = 1 \tag{7}$$

where the integration is performed over the cross section normal to y, and we have

$$e_x(x,z) = -h_z(x,z). \tag{8}$$

For the parallel-plate region, using (7), we find

$$e_x(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} = -h_z(x) \tag{9}$$

where the z-dependence everywhere is $\exp(-jk_z z)$, and $k_z = \beta$.

The other field components are readily written as

$$h_x(x,z) = \frac{1}{k_0^2 - k_z^2} \frac{\partial^2 h_z(x,z)}{\partial x \, \partial z}$$

$$e_y(x,z) = \frac{1}{jk_y} \frac{\partial e_x(x,z)}{\partial x}$$

$$h_y(x,z) = \frac{-jk_y}{k_0^2 - k_z^2} \frac{\partial h_z(x,z)}{\partial z}$$

so that

$$h_x(x) = \frac{jk_z}{k_0^2 - k_z^2} \sqrt{\frac{2}{a}} \frac{\pi}{a} \cos \frac{\pi x}{a} \tag{10}$$

$$e_y(x) = \sqrt{\frac{2}{a}} \frac{\pi}{a} \frac{1}{jk_y} \cos \frac{\pi x}{a} \tag{11}$$

$$h_y(x) = \sqrt{\frac{2}{a}} \frac{k_y k_z}{k_0^2 - k_z^2} \sin \frac{\pi x}{a}. \tag{12}$$

Again, the exponential dependence on z is omitted because it is the same for all field components.

The complete field expressions for the mode follow simply as

$$E_x(x,y,z) = V(y) e_x(x) e^{-jk_z z}$$
$$H_x(x,y,z) = I(y) h_x(x) e^{-jk_z z}$$
$$H_z(x,y,z) = I(y) h_z(x) e^{-jk_z z} \tag{13}$$

for the transverse (to y) field components, and

$$E_y(x,y,z) = Z_0 I(y) e_y(x) e^{-jk_z z}$$
$$H_y(x,y,z) = Y_0 V(y) h_y(x) e^{-jk_z z} \tag{14}$$

for the longitudinal components, where Y_0 $(=1/Z_0)$ is given by (6).

C. *The Equivalent Network for the Step Junction*

The step junction is a lossless asymmetric discontinuity, and it therefore requires three real quantities for its characterization. A useful equivalent circuit representation for that type of structure is the one shown in Fig. 4. It has been found by experience with careful measurements on a variety of step-junction discontinuities in rectangular waveguide [14] that the series reactance X_s is always very small, and that for most situations it may be safely neglected. The representation in Fig. 4(b) thus conveniently reduces to a shunt network comprised of a shunt susceptance B and a transformer with turns ratio n.

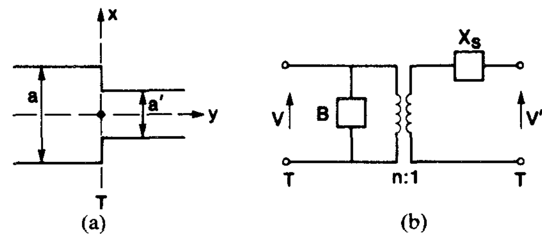


Fig. 4. The step junction and a rigorous equivalent circuit representation for it.

1) The Transformer Turns Ratio: From the equivalent circuit of Fig. 4(b), after setting $X_s = 0$, we see that the transformer turns ratio is given simply by

$$n = \frac{V}{V'}. \tag{15}$$

Consistent with (13) for the transverse electric field, we have the voltages V and V' given by

$$V(0) = \int_{-a'/2}^{a'/2} \boldsymbol{E}_t(x,0,z) \cdot \boldsymbol{e}^*(x,z) \, dx \, dz$$

$$V'(0) = \int_{-a'/2}^{a'/2} \boldsymbol{E}_t(x,0,z) \cdot \boldsymbol{e}'^*(x,z) \, dx \, dz \tag{16}$$

since $y = 0$ defines the plane of the step junction shown in Fig. 4(a) and the step is uniform along z. For simplicity, we choose the aperture transverse electric field $\boldsymbol{E}_t$ to be

$$\boldsymbol{E}_t(x,0,z) = E_a(x) e^{-jk_z z} \boldsymbol{x}_0 = A \sin \frac{\pi x}{a'} e^{-jk_z z} \boldsymbol{x}_0. \tag{17}$$

On use of (9) for the mode function (taking the exponential dependence into account) and relations (16) and (17), expression (15) for n simplifies to

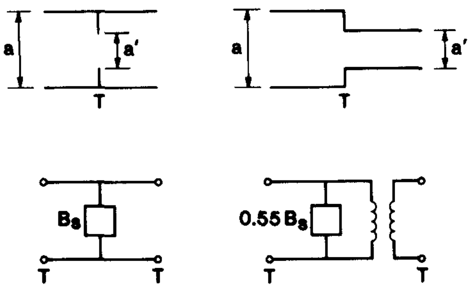
$$n = \frac{\int_{-a'/2}^{a'/2} \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \sin \frac{\pi x}{a'} \, dx}{\int_{-a'/2}^{a'/2} \sqrt{\frac{2}{a'}} \sin^2 \frac{\pi x}{a'} \, dx}$$

so that

$$n = \left[\frac{a'}{a}\right]^{3/2} \frac{4}{\pi} \frac{\cos \frac{\pi a'}{2a}}{1 - (a'/a)^2}. \tag{18}$$

2) The Shunt Susceptance: To our knowledge, an expression for the shunt susceptance for the step junction subject to the excitation shown in Fig. 2(a) is not available in the literature. By a simple additional step, however, we can adapt an available, but not widely known, result to our discontinuity of interest.

We first make use of the statement summarized in Fig. 5. To interpret that statement, we recall that the susceptance



(a) (b)

Fig. 5. Discontinuities and corresponding equivalent circuits. (a) Symmetrical planar aperture. (b) Step junction with the same a and a' dimensions as in (a).

associated with a discontinuity is proportional to its stored power, which is comprised of the evanescent higher modes trapped in its vicinity. With the *symmetrical* discontinuity in Fig. 5(a), for which the equivalent susceptance is B_s, the stored power is the same on each side of the aperture, so that each side of the aperture can be considered as contributing one-half of the susceptance. For the unsymmetrical step junction in Fig. 5(b), the electric field in the aperture will not be identical to that in the aperture of the symmetrical discontinuity, but it will not be much different from it. Assuming the two aperture fields to be the same, the contribution to the susceptance of the step junction from the side of height a (in Fig. 5(b)) is thus equal to $B_s/2$. With respect to the narrow side, of height a', the field curvature, and therefore the higher mode stored power, is much smaller than that on the other side, although it is not zero. We have found, by experience with careful measurements on step junctions of various types in rectangular waveguide [14], that to a very good approximation the susceptance of the total step junction is equal to about 0.55 B_s, as indicated in Fig. 5, where the narrower side contributes about one-tenth as much as the wide side. The actual proportion contributed by the narrower side would vary somewhat with a'/a; the ratio 0.55 must be greater than 0.50, however, which would mean no contribution at all from the narrower side, and is not likely to be greater than 0.60, which corresponds to twice the contribution implied by 0.55, and which we did not find in prior experience. The average value of 0.55 that we choose is therefore not wrong by much, if at all.

Using the rule of thumb discussed above and summarized in Fig. 5, we may seek an expression for the susceptance of the appropriate symmetric discontinuity, subject to the incident excitation seen in Fig. 2(a), and then take 0.55 times it for the susceptance of the step junction. That symmetric discontinuity is not included in the *Waveguide Handbook* [15], but it is discussed in Volume 8 of the same series [16] in the context of how Babinet's principle may be used creatively. The expression for the susceptance given there [16] for the symmetric aperture, times 0.55 so that it applies to the step junction of Fig. 5(b), and restated in terms of k_y, is

$$\frac{B}{Y_0} = 0.55 k_y \frac{2a}{\pi} \cot^2 \frac{\pi a'}{2a}. \tag{19}$$

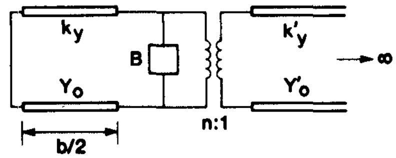


Fig. 6. Complete transverse equivalent network for open groove guide, for the excitation indicated in Fig. 2(a).

With (19), and (18) for the turns ratio, we now have available expressions for the parameters of the equivalent network for the step junction which are both simple and accurate.

D. *The Complete Transverse Equivalent Network*

In the previous sections, we have deduced all the constituent elements that comprise the transverse equivalent network for the groove guide under the excitation outlined in Fig. 2(a). The form presented in Fig. 3(b) can thus be delineated as shown in Fig. 6, where the network has been placed horizontally for convenience. The expressions for parameters B, n, and Y_0 (and therefore Y'_0) are given respectively by (19), (18), and (6). The form of the network and the expressions for its constituents are seen to be eminently simple, and yet they characterize the structure very accurately.

Once the network in Fig. 6 becomes available, the determination of the dispersion relation for the lowest mode becomes an essentially trivial task. We choose a reference plane T somewhere, say just to the left of susceptance B, and apply the transverse resonance relation

$$\overleftarrow{Y}(T) + \overrightarrow{Y}(T) = 0$$

yielding

$$j \cot k_y \frac{b}{2} = \frac{1}{n^2} \frac{Y'_0}{Y_0} + j \frac{B}{Y_0}. \tag{20}$$

We next recognize, as have all the previous authors, that the outer regions, with wavenumber k'_y, are transversely evanescent, so that

$$k'_y = -j|k'_y| \tag{21}$$

and the transmission line extending to infinity is below cutoff. Using (21), we note from (6) that

$$\frac{Y'_0}{Y_0} = j \frac{k_y}{|k'_y|} \tag{22}$$

so that (20) becomes

$$\cot k_y \frac{b}{2} = \frac{1}{n^2} \frac{k_y}{|k'_y|} + k_y \left[0.55 \frac{2a}{\pi} \cot^2 \frac{\pi a'}{2a} \right] \tag{23}$$

where (19) was employed and where it is recognized that n is a geometry-dependent constant, independent of the wavenumbers.

The transverse wavenumbers k_y and $|k_y'|$ are related to the longitudinal wavenumber $k_z = \beta$, and the frequency, via k_0, by

$$k_0^2 = \beta^2 + \left[\frac{\pi}{a}\right]^2 + k_y^2$$

$$k_0^2 = \beta^2 + \left[\frac{\pi}{a'}\right]^2 - |k_y'|^2 \tag{24}$$

for the central and outer regions, respectively, so that by subtraction we have

$$k_y'^2 = \left[\frac{\pi}{a'}\right]^2 - \left[\frac{\pi}{a}\right]^2 - k_y^2. \tag{25}$$

Relation (25) may then be placed into (23) to yield an explicit expression for k_y, which is seen to be dependent on geometry only, and therefore independent of frequency. The cutoff wavenumber k_c for the groove guide then becomes simply

$$k_c^2 = k_y^2 + \left[\frac{\pi}{a}\right]^2. \tag{26}$$

The early first-order solution derived by various authors [4]–[7], [9] corresponds precisely to the first two terms in (23). The third term in (23) represents a particularly simple and convenient way to take into account the influence of all the higher modes, which the first-order solution admittedly neglects. The higher modes have alternatively been included [4]–[7] by a mode-matching technique which yields a dispersion relation in matrix form involving a series of linear equations. The truncation of the matrix depends on the number of higher modes taken into account, but complexity increases rapidly and the added contribution from the next one or two modes has been found to be small [6], [7]. On the other hand, the third term in (23) eliminates the need for such matrix calculations, and permits one to obtain highly accurate final results with very little extra calculational effort. Comparison with careful measurements from the literature [4], [5] is made in the next section.

One final remark may be helpful. Confusion exists in the literature regarding what is meant by the "transverse resonance solution." It is interpreted by some to mean only the first-order solution, in contrast to the mode-matching procedure which is viewed as "exact." The mode-matching process is exactly *phrased*, but it becomes necessarily approximate once the matrix is truncated. Its accuracy must then be examined relative to that of other approximate methods, such as the one leading to (23). Furthermore, *all* of these procedures are transverse resonance solutions; the transverse resonance method can be employed to various degrees of accuracy, and may or may not be phrased in direct network terms, as it has been here.

III. Numerical Results

The principal aim of this section, in its presentation and discussion of numerical results, is to *verify the accuracy* of the new theoretical results presented in Section II, as embodied in dispersion relation (23) and the simple transverse equivalent network shown in Fig. 6. Toward this end, we present in Section III-A below a comparison between our theoretical numbers and the careful *experimental* results of Nakahara and Kurauchi [4], [5]. A secondary purpose is the presentation of additional numerical results, showing parametric dependences which have not been given previously. Those additional numerical data appear in Section III-B.

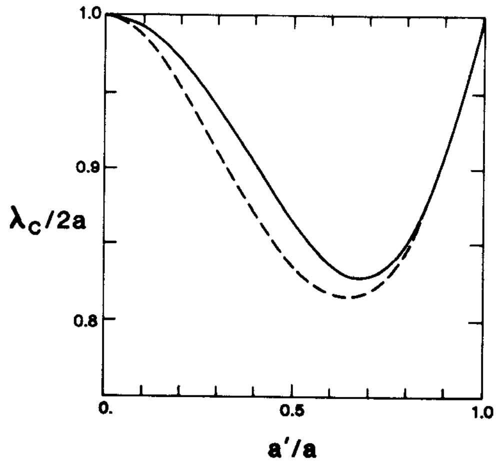


Fig. 7. Theoretical values of $\lambda_c/2a$, where λ_c is the cutoff wavelength, versus relative width a'/a of groove guide. In these calculations, $b/a = 0.400$. The solid curve represents our improved theory, and the dashed curve corresponds to the first-order theory of Nakahara and Kurauchi [4], [5].

A. Comparison with Measurements

Nakahara and Kurauchi (referred to hereafter as N–K) have presented in English their early comprehensive study of groove guide in the form of two papers, which bear the same title and overlap each other substantially [4], [5]. The two figures with which we shall make comparisons are identical in each, but the figure numbers are different for one of them. One of these figures presents first-order theoretical numbers for the variation of the cutoff wavelength λ_c ($= 2\pi/k_c$) with the relative width of the outer sections. We will superimpose our more accurate theoretical numbers on their plot to indicate that the corrections introduced by our theory can be significant over a large range of parameter values. The other figure contains their experimental data together with first-order theoretical values for a number of guide cross sections. We shall superimpose our theoretical values on their plots to show the improvement provided by our theory.

It should be pointed out that the symbols used for various dimensions are different in our paper and in those by N–K. Their b and c are our a and a', and their $2l$ is our b; the coordinate systems are identical. In the discussion that follows, we employ our notation.

We first make comparison with the N–K first-order theoretical values for $\lambda_c/2a$ versus a'/a, which appear as Fig. 8 in both of their papers. That curve is duplicated in Fig. 7 here as the dashed curve; the solid curve corresponds to the solution of (23) and then use of (26). It is seen that the two curves are almost identical for $a'/a > 0.8$. In that

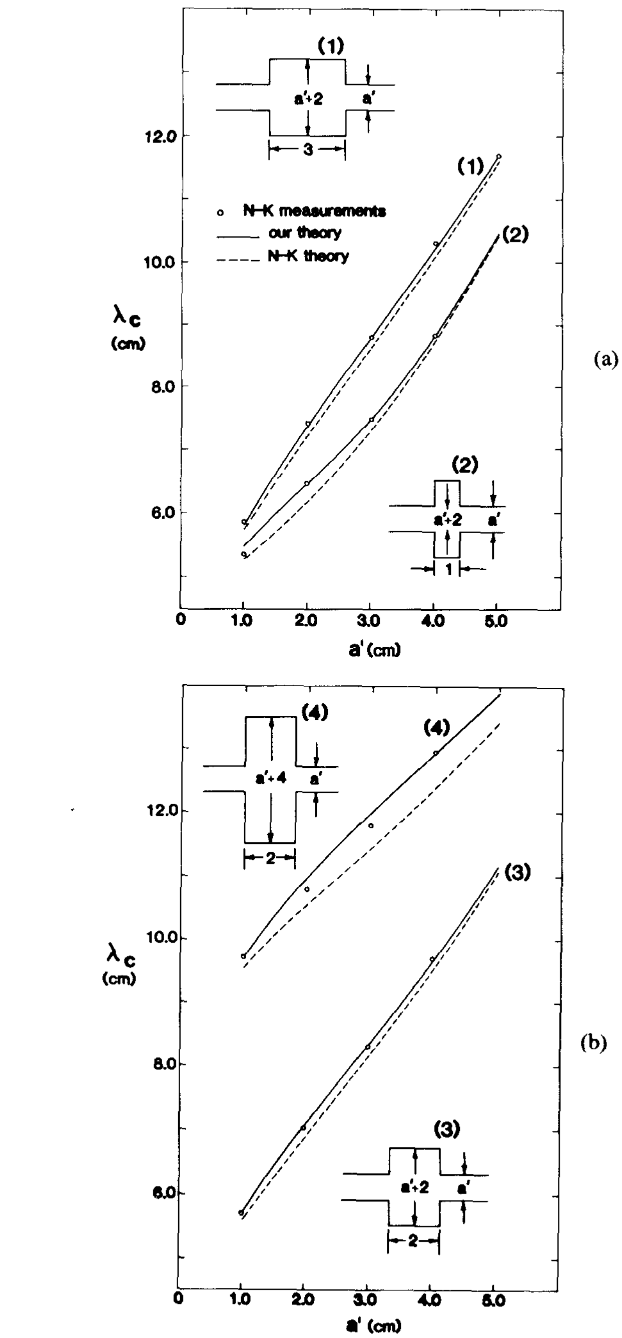


Fig. 8. Comparisons between measured and theoretical values of the cutoff wavelength λ_c for groove guides of various cross sections. The solid lines represent our improved theory, the dashed curves are the first-order theoretical values, and the points are the measured results of Nakahara and Kurauchi [4], [5]. The insets indicate the cross-section geometries for each measured point, where the numbers are in centimeters.

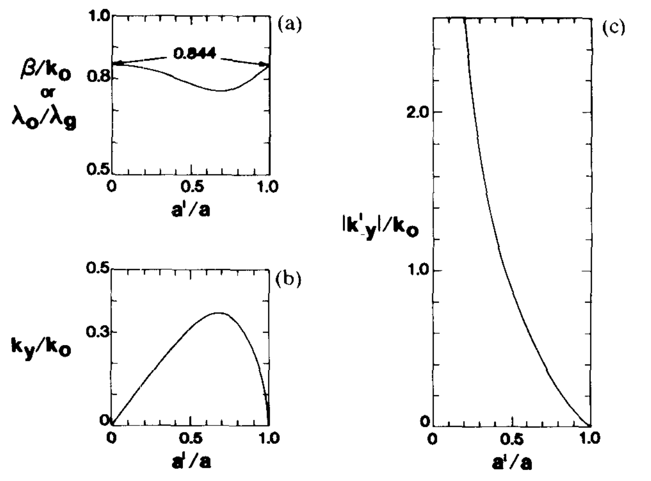


Fig. 9. The variations of β/k_0, k_y/k_0, and $|k_y'|/k_0$ as a function of groove guide relative width a'/a, with $a = 1.00$ cm and $b/a = 0.400$, at a frequency of 28.0 GHz. β is the propagation wavenumber, and k_y and $|k_y'|$ are transverse wavenumbers.

range, the value of susceptance B/Y_0 is relatively small, so that its neglect in the first-order theory is justified. For smaller a'/a values, however, the discontinuity due to the step junction is more pronounced, and the susceptance contribution becomes more important, as is evident from Fig. 7.

In their papers, N–K make the following interesting observation. In the limit for which $a'/a = 1$, the modal configuration becomes that of the TE_1 mode in parallel-plate waveguide of width a. In the other limit, $a'/a = 0$, one obtains the TE_{10} mode in rectangular waveguide of width a. In both limits, the value of $\lambda_c/2a$ should equal unity, as found in Fig. 7. The curves are therefore exact in the limits and reasonably accurate elsewhere. The solid curve, corresponding to our theory, should everywhere be more accurate than the first-order theory, which is the one that is usually employed and is represented by the dashed line.

In [5, fig. 9] and [4, fig. 10], N–K present the results of careful *measurements* on a variety of groove guides. They give the measured values of λ_c as a function of a' for groove guides of different cross sections, and they show how these values compare with curves obtained using first-order theory. All of those data, plus our theoretical numbers, are contained in Fig. 8(a) and (b) presented here; the first-order theory is represented by dashed lines, our more accurate theory by solid lines, and the measured data as discrete points. The cross sections corresponding to each set of curves are shown as insets.

It is seen that our theoretical curves agree very well with the measured values in almost all cases. On the other hand, the first-order theoretical values are systematically somewhat below both our theory and the measured data. It appears, therefore, that the first-order theory represents a rather good approximation, considering its simplicity, and that the *new theory* using (23) is indeed *significantly more accurate*.

B. *Additional Numerical Results*

A few additional numerical results are presented here which illustrate the dependences of the propagation (longitudinal) wavenumber β $(= k_z)$ and the transverse wavenumbers k_y and $|k_y'|$ upon the two geometric ratios a'/a and b/a. Our improved theory is employed in determining these numbers. The propagation wavenumber β varies with frequency, but the transverse wavenumbers do not. For convenience, however, all wavenumbers in the discussion

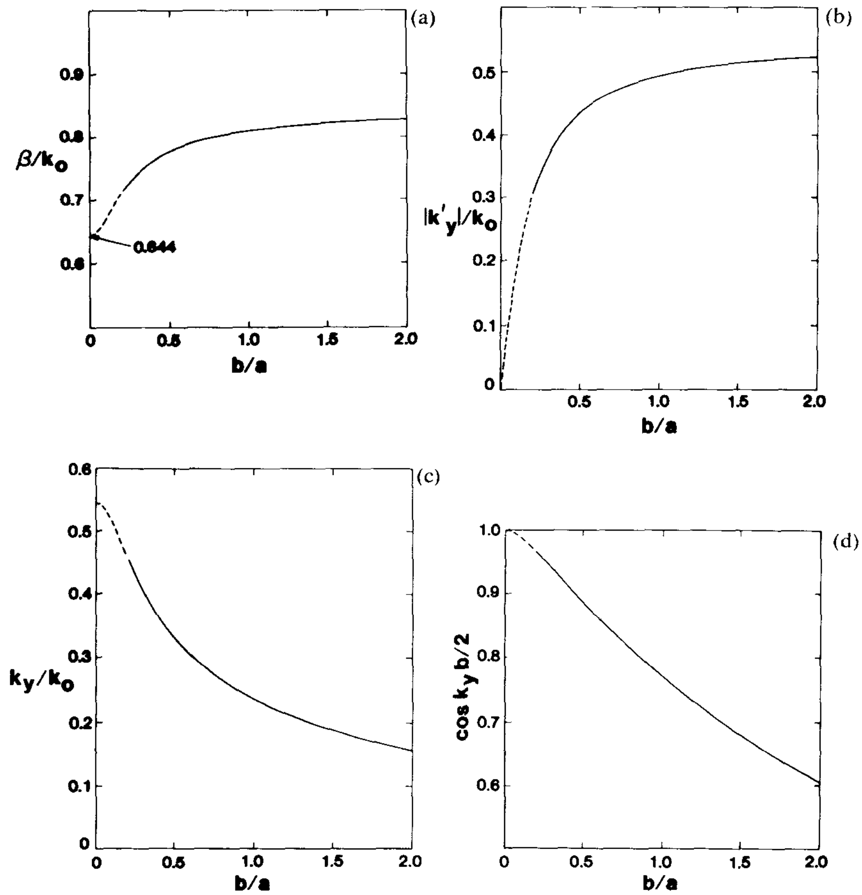


Fig. 10. Variation of certain wavenumbers with groove guide aspect ratio b/a, with $a = 1.00$ cm and $a'/a = 0.700$, at a frequency of 28.0 GHz. (a) Propagation wavenumber β, (b) transverse wavenumber $|k'_y|$, (c) transverse wavenumber k_y, and (d) the ratio of the exciting field at the step junction to that at the groove center.

below are normalized to the free-space wavenumber k_0, corresponding to a frequency of 28.0 GHz, so that $k_0 = 5.87/\text{cm}$ and $\lambda_0 = 1.07$ cm. The width a is also specified as 1.00 cm in these calculations.

We consider first the variations with a'/a. It was remarked above that at the two limit points $a'/a = 0$ and $a'/a = 1$ the groove guide structure degenerates respectively into a rectangular waveguide of width a and a parallel-plate waveguide of width a, with TE_{10}-mode excitation in the former and TE_1-mode excitation in the latter. Thus, for $a'/a = 0$, we would expect $k_y = 0$, $|k'_y| = \infty$, and $\beta/k_0 = 0.844$. For $a'/a = 1$, we should find $k_y = 0$, $|k'_y| = 0$, and $\beta/k_0 = 0.844$ again.

The dependences of β/k_0, k_y/k_0, and $|k'_y|/k_0$ as a function of a'/a are shown in Fig. 9(a)–(c), respectively. For these curves, $b = 0.400$ cm. It is seen from Fig. 9(a) that the value of β/k_0 does not vary by more than about ± 5 percent over the whole range of a'/a, with the highest values at the end points and the lowest value near $a'/a = 0.7$. Comparison of Fig. 9(b) with Fig. 7 shows that k_y/k_0 and $\lambda_c/2a$ vary qualitatively with a'/a in much the same way, but in inverse fashion; the deviation from the end points is greatest near $a'/a = 0.7$ for both. The biggest variation appears in Fig. 9(c) for $|k'_y|$, since it must vary from zero to infinity between the end points. It is clear, then, that *greater confinement* of the fields to the central region is achieved simply by reducing the a'/a ratio, and that the value of β/k_0 $(= \lambda_0/\lambda_g)$ is changed little in the process.

Finally, we consider the variation of these wavenumbers with b/a, the *aspect ratio* of the groove guide. In the limit $b/a = 0$, the groove guide becomes a parallel-plate guide of width a', supporting the TE_1 mode. Thus, we would find $|k'_y| = 0$, $k_x = \pi/a'$, $\beta/k_0 = 0.644$ at 28.0 GHz, and k_y/k_0 taking on the limiting value 0.547. The behavior of the wavenumbers with b/a, for $a'/a = 0.700$, $a = 1.00$ cm, and a frequency of 28.0 GHz, are presented in Fig. 10(a)–(c).

The end-point performance is as anticipated, but it is also seen that β/k_0 and $|k'_y|/k_0$ do not change much at the larger values of b/a. (Of course, even small changes in $|k'_y|$ may be significant since that wavenumber appears in an exponential.) The variation of k_y/k_0 with b/a seems strong, but its significance is better appreciated when the function $\cos(k_y b/2)$ is evaluated. That function indicates the ratio of the dominant transverse mode field at the step junction to that at the groove center, and is shown in Fig. 10(d). It is therefore seen that even though k_y is larger when b is smaller, the product $k_y b$ decreases as b is reduced, and the actual field variation with y in the central region becomes less.

IV. Conclusions

A new solution is presented here for the propagation properties of the dominant mode in open, but nonradiating, groove guide. This solution is a significant improvement over the ones contained in the literature, in that it permits highly accurate results for the propagation constant even though it is simple and in closed form. Furthermore, the solution is accurate over a wide range of geometric parameter values.

The dispersion relation for the propagation properties of the dominant mode is (23), and it corresponds to the lowest resonance of the transverse equivalent network given in Fig. 6. Expressions for all the elements of this network have been derived in such a way that they are in simple closed form, and yet they are accurate representations. These derivations are presented in detail in Section II. The essentially new contribution involves the characterization of the step-junction discontinuity. Previous representations either neglected the higher mode content or accounted for the higher modes in a slowly convergent manner. Our contribution is to deduce a simple closed-form expression for that discontinuity which accurately accounts for the higher modes. As a result, the complete transverse equivalent network contains all elements in simple closed form, leading to a dispersion relation which is correspondingly simple.

The most important numerical discussion in Section III is that which relates to Fig. 8; comparisons are presented in Fig. 8 of measured and theoretical results for four different sets of geometries. The careful measurements were taken by Nakahara and Kurauchi [4], [5], and the theoretical values are of two types: first-order results also presented by Nakahara and Kurauchi [4], [5], and numbers obtained using our improved theory. The comparisons show clearly that our theory is significantly more accurate, and agrees very well with the measurements.

Other numerical calculations are presented in Section III to demonstrate other performance features. For example, a comparison in Fig. 7 involving λ_c shows that the approximate solution can yield rather good results when $a'/a > 0.8$, where the step-junction discontinuity is small, but that significant errors can arise for other values of a'/a. It is also shown that the confinement of fields in the cross section to the central region can be controlled dramatically by simply adjusting the a'/a ratio, and that the longitudinal wavenumber β is not strongly affected by such an adjustment. The dependences of the wavenumbers on the aspect ratio b/a are also discussed.

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